Vol. XLV. JUNE, 1901.

AMERICAN SOCIETY OF CIVIL ENGINEERS.

INSTITUTED 1852.

TRANSACTIONS.

NOTE.—This Society is not responsible, as a body, for the facts and opinions advanced in any of its publications.

No. 893.

THE ADJUSTMENT OF A TRANSIT SURVEY AS COMPARED WITH THAT OF A COMPASS SURVEY.

By CHARLES L. CRANDALL, M. Am. Soc. C. E.

TO BE PRESENTED FEBRUARY 6TH, 1901.

WITH DISCUSSION.

COMPASS SURVEY.

The method of adjustment of the latitudes and departures of a compass survey by Dr. Bowditch's rule, distributing the error in latitude proportionately to the lengths of the sides, and similarly for the departures, is familiar to all civil engineers and surveyors.

The fundamental principles upon which it is based, and the assumption which was made to render the rule simple, are not so generally known.

The main steps are readily traced. The latitude equation is

$$L = l \cos. B \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where L = latitude, l = length of side, and B = bearing.

If this equation be differentiated first with respect to l and then with respect to B, it will give the effect upon the latitude of a small change in distance, or of a small change in bearing.

Thus, $d\,L' = d\,l \cos.\ B.$

$$d\,L'' = -\,l \sin.\ B\,d\,B.$$

The uncertainty, or error as it is usually called, in latitude, due to the combined effect of an uncertainty in distance and also in bearing, is equal to the square root of the sum of the squares of the effects due to each separately, where positive and negative errors are equally probable. Thus, if r_l = the error in distance, replacing $d\,l$, r_B = the error in bearing, replacing $d\,B$, the corresponding error in latitude squared,

$$r_L^2 = r_l^2 \cos.^2 B + l^2\, r_B^2 \sin.^2 B \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

For the departure,

$$D = l \sin.\ B \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Differentiating and substituting as above,

$$r_D^2 = r_l^2 \sin.^2 B + l^2\, r_B^2 \cos.^2 B \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Dr. Bowditch assumed,

$$r_l^2 = r_B^2\, l^2 = l \times \text{constant} = l\,C \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

which reduces equations (2) and (4) to

$$r_L^2 = r_D^2 = l\,C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

i. e., the squares of the uncertainties, or probable errors, in latitude and in departure, are proportional to the lengths of the sides.

In the Method of Least Squares it is shown that the most probable corrections will be those the sum of whose squares is a minimum after each correction has been divided by its probable error; *i. e.*, if the corrections in latitude for the different sides be denoted by v_1, v_2, $v_3 \ldots$, we shall have

$$\frac{v_1^2}{l_1} + \frac{v_2^2}{l_2} + \frac{v_3^2}{l_3} + \ldots = \text{minimum.}$$

Differentiating,

$$\frac{v_1}{l_1}\,d\,v_1 + \frac{v_2}{l_2}\,d\,v_2 + \frac{v_3}{l_3}\,d\,v_3 + \ldots = 0 \ldots\ldots\ldots\ldots\ldots (7)$$

Since the sum of the corrections must equal the total error in latitude with its sign changed, a constant,

$$v_1 + v_2 + v_3 + \ldots = k.$$

Differentiating,

$$d\,v_1 + d\,v_2 + d\,v_3 + \ldots = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Comparing (7) and (8), and remembering that each must hold whatever the number of sides, or v's, we have

$$\frac{v_1}{l_1} = \frac{v_2}{l_2} = \frac{v_3}{l_3} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

i. e., the corrections in latitude are proportional to the lengths of the sides, according to the old rule.

The same demonstration will hold for the corrections in departure.

The interesting point, in this connection, is the assumption—Equation (5)—made by Dr. Bowditch that the probable error in chaining equals the probable error in bearing (in π-measure) multiplied by the distance, and increases as the square root of the distance.

The inaccuracy of chaining is usually spoken of as proportional to the distance, but in comparing the different lines of a survey measured with the same chain, the square root of the distance as assumed above is more nearly correct. Assuming an uncertainty which would amount to 1 ft. in 500, which is not unreasonable for compass work, we would have from (5)

$$r_{500} = 1 = \sqrt{500\ C}\ ;\ \text{or}\ C = 0.0447,$$

from which $r_l = 0.0447\sqrt{l}$; $r_B\,(\text{in minutes}) = \dfrac{0.0447}{\sin.\ 1'\sqrt{l}}$

Substituting for different distances gives the results shown in Table No. 1.

TABLE No. 1.

Distance, in feet.	Uncertainty in chaining, in feet.	Uncertainty in bearing.
10	0.14	0° 48′
50	0.32	0 21
100	0.45	0 15
500	1.00	0 07
1 000	1.41	0 05
2 000	2.00	0 03

An examination of Table No. 1 will show that the assumption is fairly reasonable, although it gives too great weight to the bearings of long lines, and perhaps too small to those of very short lines.

The method is thus satisfactory as to accuracy and simplicity when applied to the adjustment of a compass survey.

Transit Survey.

In a transit survey proper, no bearings are taken except as a check, but the horizontal angles between the lines are measured. In computing latitudes and departures, or co-ordinates, a meridian is assumed, and the bearings or azimuths are computed from the

horizontal angles. The bearing of the first line from the meridian involves only one angle; that of the second, two angles; etc., one-half way around the figure; the bearings for the other half being more accurately computed by working backward from the meridian.

Thus, with the following notes:

Sta. 1.	Deflection, $M_1 = 92° 0' R.$	Distance, $l_1 = 236$ ft.
2.	$M_2 = 46 \ 0 \ R.$	$l_2 = 244$
3.	$M_3 = 82 \ 0 \ R.$	$l_3 = 324$
4.	$M_4 = 51 \ 57 \ R.$	$l_4 = 183$
5.	$M_5 = 88 \ 0 \ R.$	$l_5 = 437$

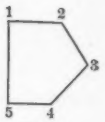

FIG. 1.

Assume 5 — 1 as a meridian (Fig. 1).

Bearing, $B_1 = 180 - M_1 = \text{S.}88° 00' \text{E.}$

$B_2 = B_1 - M_2 = 180 - (M_1 + M_2) = \text{S.}42 \ 00 \ \text{E.}$

$B_3 = M_3 - B_2 = M_1 + M_2 + M_3 - 180° = \text{S.}40 \ 00 \ \text{W.}$

$= 180° - (M_5 + M_4) = \text{S.}40 \ 03 \ \text{W.}$

$B_4 = M_4 = \text{N.}88 \ 00 \ \text{W.}$

The discrepancy in angle work is thus thrown into the difference between the bearings of the two lines meeting at 3, and would not be properly distributed by the compass method of adjustment. If the deflections were first adjusted to close the figure, the compass survey method would allow the inaccuracy of the chaining to change the adjusted angles without giving proper weight to the accuracy of angle measurement.

There is thus no reason to assume, as appears to have been almost universally done, that because the Dr. Bowditch rule gives a close adjustment for a compass survey, it will do so for a transit survey.

Theoretically, there are three equations which must be satisfied in adjusting the observed quantities:

(*a*) The sum of the interior angles must equal twice as many right angles as the figure has sides, less four.

(*b*) The algebraic sum of the latitudes must equal zero.

(*c*) The algebraic sum of the departures must equal zero.

The Method of Least Squares requires that the corrections must be such as to satisfy the above conditions, and make the sum of their squares, each square being divided by the square of its probable error, a minimum. Since each bearing must be expressed in terms of the measured angles, and the number of measured angles in each

varies from one to one-half the number of sides, the labor involved is too great for the adjustment of an ordinary transit survey.

In ordinary work the probable error of an angle need not exceed 1 minute, if care is taken in setting over the points and in plumbing the flag poles, using tacks on the stakes for all lines of less than 300 ft., swinging without delay from the back sight to the front sight, and lining in a "range" point to swing from for all lines of less than 50 ft. With these precautions, the probable error need not increase with the shortness of the line, as with the compass.

On very rough ground, or in going through brush, where the flag pole is partly hidden, it may be difficult to keep the probable error below 2 minutes; while, for careful work, the probable error can be readily kept within $\frac{1}{2}$ minute.

For good work the length of sight should be limited to about 1 200 ft.

Ordinarily, it will be more difficult to measure distances to 1:500 than angles to minutes, while an accuracy of 1 : 1 000 is seldom reached except on level ground or for city work.

The accuracy of angle work is thus considerably greater than that of chaining, 1 minute in angle giving 0.15 ft. in 500 as compared with 1 ft. in chaining; or, 0.5 minute, 0.15 ft. in 1 000, as compared with the 1 ft. due to the more accurate chaining.

On this account, it will be admissible to first adjust the angles to close the figure, then compute the bearings or azimuths and assume them to be correct in the final adjustment.

This leaves only the two conditions:

(*b*) Sum of latitudes equal zero.

(*c*) Sum of departures equal zero.

That is,

$$\left.\begin{aligned} l_1 \cos.\ B_1 + l_2 \cos.\ B_2 + l_3 \cos.\ B_3 + = 0 \\ l_1 \sin.\ B_1 + l_2 \sin.\ B_2 + l_3 \sin.\ B_3 + = 0 \end{aligned}\right\} \quad \ldots\ldots\ldots (10)$$

where the total corrections are to be applied on the basis of inaccuracy in chaining.

In order to derive these corrections on the basis of least squares, denote the observed distances by M_1, M_2, M_3, . . . , and the required corrections by v_1, v_2, v_3, . . . The corrected distances will be

$$l_1 = M_1 + v_1,\ l_2 = M_2 + v_2,\ l_3 = M_3 + v_3 \ldots$$

Substituting in (10),

$$\left.\begin{aligned}(M_1 + v_1)\cos.\ B_1 + (M_2 + v_2)\cos.\ B_2 + \ldots &= 0\\ (M_1 + v_1)\sin.\ B_1 + (M_2 + v_2)\sin.\ B_2 + \ldots &= 0, \text{ or}\\ v_1\cos.\ B_1 + v_2\cos.\ B_2 + v_3\cos.\ B_3 + \ldots + q_1 &= 0\\ v_1\sin.\ B_1 + v_2\sin.\ B_2 + v_3\sin.\ B_3 + \ldots + q_2 &= 0\end{aligned}\right\} \ldots\ (11)$$

where

$q_1 = M_1\cos.\ B_1 + M_2\cos.\ B_2 + M_3\cos.\ B_3 + \ldots =$ error in latitude,

$q_2 = M_1\sin.\ B_1 + M_2\sin.\ B_2 + M_3\sin.\ B_3 + \ldots =$ error in departure.

For convenience, change (11) to

$$\left.\begin{aligned}v_1\frac{L_1}{l_1} + v_2\frac{L_2}{l_2} + v_3\frac{L_3}{l_3} + \ldots + q_1 &= 0\\ v_1\frac{D_1}{l_1} + v_2\frac{D_2}{l_2} + v_3\frac{D_3}{l_3} + \ldots + q_2 &= 0\end{aligned}\right\} \ldots\ldots\ldots (12)$$

where $L_1, L_2, \ldots, D_1, D_2, \ldots$, denote latitudes and departures.

Aside from (12) we must also have $v_1^2 + v_2^2 + v_3^2 + \ldots$ a minimum, for equal accuracy in chaining the different lines; or, if the accuracy in chaining be assumed to decrease as the square root of the distance,

$$\frac{v_1^2}{l_1} + \frac{v_2^2}{l_2} + \frac{v_3^2}{l_3} + \ldots = \text{minimum}.$$

Differentiating,

$$\frac{v_1}{l_1}dv_1 + \frac{v_2}{l_2}dv_2 + \frac{v_3}{l_3}dv_3 + \ldots = 0 \ldots\ldots\ldots\ldots (13)$$

If the dv's were independent, their coefficients would have to separately equal zero for (13) to hold true. But the v's are connected by (12). Differentiating (12),

$$\left.\begin{aligned}\frac{L_1}{l_1}dv_1 + \frac{L_2}{l_2}dv_2 + \frac{L_3}{l_3}dv_3 + \ldots &= 0\\ \frac{D_1}{l_1}dv_1 + \frac{D_2}{l_2}dv_2 + \frac{D_2}{l_2}dv_2 + \ldots &= 0\end{aligned}\right\} \ldots\ldots\ldots\ldots (14)$$

If we eliminate a dv in (13) for each of (14), the remaining dv's will be independent, and hence their coefficients will separately equal zero. This elimination is best done by the method of indeterminate multipliers, *i. e.*, multiply the first of (14) by x, the second by y, and (13) by -1, and add,

$$\left(x\frac{L_1}{l_1} + y\frac{D_1}{l_1} - \frac{v_1}{l_1}\right)dv_1 + \left(x\frac{L_2}{l_2} + y\frac{D_2}{l_2} - \frac{v_2}{l_2}\right)dv_2 + \ldots = 0 \ldots (15)$$

Now give x and y such values that the coefficients of two of the dv's will be zero, thus eliminating them. The remaining dv's will be inde-

pendent, and hence their coefficients will be zero. This makes the coefficients of all the dv's in (15) equal zero, giving

$$\left.\begin{aligned} x\frac{L_1}{l_1}+y\frac{D_1}{l_1}-\frac{v_1}{l_1}&=0\\ x\frac{L_2}{l_2}+y\frac{D_2}{l_2}-\frac{v_2}{l_2}&=0\\ x\frac{L_3}{l_3}+y\frac{D_3}{l_3}-\frac{v_3}{l_3}&=0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (16)$$

To find x and y, multiply the first of (16) by L_1, the second by L_2, . . . , and add; then by D_1, D_2, . . . , and add; then by (12)

$$\left.\begin{aligned} \Sigma\left(\frac{L^2}{l}\right)x+\Sigma\left(\frac{L\,D}{l}\right)y+q_1&=0\\ \Sigma\left(\frac{L\,D}{l}\right)x+\Sigma\left(\frac{D^2}{l}\right)y+q_2&=0 \end{aligned}\right\} \quad \ldots\ldots\ldots (17)$$

Σ, as usual, denoting the summation of a number of similar terms,

$$i.\,e.,\ \Sigma\left(\frac{L^2}{l}\right)=\frac{L_1^2}{l_1}+\frac{L_2^2}{l_2}+\frac{L_3^2}{l_3}+.$$

Solving,

$$\left.\begin{aligned} x&=\frac{q_2\,\Sigma\left(\frac{L\,D}{l}\right)-q_1\,\Sigma\left(\frac{D^2}{l}\right)}{\Sigma\left(\frac{D^2}{l}\right)\Sigma\left(\frac{L^2}{l}\right)-\Sigma\left(\frac{L\,D}{l}\right)^2}\\ y&=\frac{q_1\,\Sigma\left(\frac{L\,D}{l}\right)-q_2\,\Sigma\left(\frac{L^2}{l}\right)}{\Sigma\left(\frac{D^2}{l}\right)\Sigma\left(\frac{L^2}{l}\right)-\Sigma\left(\frac{L\,D}{l}\right)^2} \end{aligned}\right\} \quad \ldots\ldots\ldots (18)$$

Substituting in (16),

$$\left.\begin{aligned} v_1&=L_1x+D_1y\\ v_2&=L_2x+D_2y\\ &\ldots\ldots\ldots\ldots \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (19)$$

Adding, $\Sigma\,(v)=x\,\Sigma\,(L)+y\,\Sigma\,(D),=0$, nearly. . . . . (20)

Also,

$$\left.\begin{aligned} v_{L_1}=\frac{L_1}{l_1}v_1&=\frac{L_1^2}{l_1}x+\frac{L_1\,D_1}{l_1}y\\ v_{L_2}=\frac{L_2}{l_2}v_2&=\frac{L_2^2}{l_2}x+\frac{L_2\,D_2}{l_2}y\\ &\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} \quad \ldots\ldots\ldots (21)$$

$$\left.\begin{aligned} v_{D_1}=\frac{D_1}{l_1}v_1&=\frac{L_1\,D_1}{l_1}x+\frac{D_1^2}{l_1}y\\ v_{D_2}=\frac{D_2}{l_2}v_2&=\frac{L_2\,D_2}{l_2}x+\frac{D_2^2}{l_2}y\\ &\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} \quad \ldots\ldots\ldots (22)$$

with, by (17), $\Sigma\,(v_L)=-q_1$ and $\Sigma\,(v_D)=-q_2$. . . . . (23)

Practical Applications.

Having adjusted the transit work, or the angles, to geometrical consistency, by distributing the small error of closure equally among the angles, and having computed the co-ordinates, Formulas (18) to (23) give the method of adjustment of the co-ordinates and of the measured sides by least squares. In deriving these formulas the assumption made by Dr. Bowditch, that the inaccuracy in chaining increases as the square root of the distance, has been retained. The application is quite simple, as all the terms are made up from the latitudes, departures and distances already entered upon the co-ordinate sheet, while the multiplications and divisions can be made with the ordinary slide rule, retaining only three figures.

In order to equalize numbers, so as to retain the same number of decimal places throughout, $100\,l$ is used in place of l in (18), and the change carried through, as follows:

$$\left.\begin{aligned} x' &= \frac{q_2 \Sigma\left(\frac{L\,D}{100\,l}\right) - q_1 \Sigma\left(\frac{D^2}{100\,l}\right)}{\Sigma\left(\frac{D^2}{100\,l}\right)\Sigma\left(\frac{L^2}{100\,l}\right) - \Sigma\left(\frac{L\,D}{100\,l}\right)^2} \\ y' &= \frac{q_1 \Sigma\left(\frac{L\,D}{100\,l}\right) - q_2 \Sigma\left(\frac{L^2}{100\,l}\right)}{\Sigma\left(\frac{D^2}{100\,l}\right)\Sigma\left(\frac{L^2}{100\,l}\right) - \Sigma\left(\frac{L\,D}{100\,l}\right)^2} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (18)'$$

$$\left.\begin{aligned} v_1 &= \frac{L_1}{100}x' + \frac{D_1}{100}y' \\ v_2 &= \frac{L_2}{100}x' + \frac{D_2}{100}y' \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)'$$

$$\Sigma(v) = x'\,\Sigma\frac{(L)}{100} + y'\,\Sigma\frac{(D)}{100} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)'$$

$$\left.\begin{aligned} v_{L_1} &= \frac{L_1}{l_1}v_1 = \frac{L_1^2}{100\,l_1}x' + \frac{L_1 D_1}{100\,l_1}y' \\ v_{L_2} &= \frac{L_2}{l_2}v_2 = \frac{L_2^2}{100\,l_2}x' + \frac{L_2 D_2}{100\,l_2}y' \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)'$$

$$\left.\begin{aligned} v_{D_1} &= \frac{D_1}{l_1}v_1 = \frac{L_1 D_1}{100\,l_1}x' + \frac{D_1^2}{100\,l_1}y' \\ v_{D_2} &= \frac{D_2}{l_2}v_2 = \frac{L_2 D_2}{100\,l_2}x' + \frac{D_2^2}{100\,l_2}y' \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (22)'$$

$$\Sigma(v_L) = -q_1, \qquad \Sigma(v_D) = -q_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)'$$

With the following notation:

x' and y' may be called auxiliaries;

q_1 = error in latitude;

q_2 = error in departure;

$L_1, L_2, L_3, \ldots,$ = latitudes of sides $l_1, l_2, l_3, \ldots$;

$D_1, D_2, D_3, \ldots,$ = departures of sides $l_1, l_2, l_3, \ldots$;

Σ denotes the summation of a number of similar terms;

$v_1, v_2, v_3, \ldots,$ = corrections to sides $l_1, l_2, l_3, \ldots$;

$v_{L_1}, v_{L_2}, v_{L_3}, \ldots, v_{D_1}, v_{D_2}, v_{D_3}, \ldots,$ = corrections to latitudes and departures, respectively.

The application of the method is best shown by a problem.

TABLE No. 2.—PROBLEM FROM DATA PREVIOUSLY GIVEN.

Station.	Deflection to the *R.*	Correction.	Azimuth.	Dist.	Latitude, *L.*		Departure, *D*		$\frac{L^2}{100\,l}$	$\frac{D^2}{100\,l}$	$\frac{L\,D}{100\,l}$
					+	−	+	−			
1.....	92° 00′	+0.6′	92° 0.6′	236	.....	8.3	235.9	.....	0.0	2.36	−0.08
2.....	46 00	0.6	138 1.2	244	.....	181.4	163.2	.....	1.35	1.09	−1.21
3.....	82 00	0.6	220 1.8	324	.....	248.1	.....	208.4	1.90	1.38	+1.60
4.....	51 57	0.6	271 59.4	183	6.3	.....	.....	182.9	0.0	1.83	−0.06
5.....	88 00	0.6	360 0.0	437	437.0	.....	.....	0.0	4.37	0.0	0.0
	359 57	3.0	Totals...	.....	443.3	437.8	399.1	391.3	7.62	6.66	+0.25
					437.8	.....	391.3				
				q_1	=+5.5		q_2 =+7.8				

The observed angles foot up 3 minutes too small. To each angle 0.6 minute is added, and the azimuths computed with 5–1 as an axis. The latitudes and departures are computed and added as usual, giving $q_1 = +5.5$, and $q_2 = +7.8$. The three columns at the right are used in finding x' and y'. To fill them out, set the slide rule to latitude divided by distance ($L \div l$) for each horizontal line and multiply by one-hundredth of the latitude (0.01 L) for the corresponding term in the $\frac{L^2}{100\,l}$ column; with the same settings multiply by one-hundredth of the departure for the corresponding terms in the $\frac{L\,D}{100\,l}$ column. Then set to departure divided by distance and multiply by one-

hundredth of the departure for the $\frac{D^2}{100\,l}$ column. Add the three columns for $\Sigma\left(\frac{L^2}{100\,l}\right)$, $\Sigma\left(\frac{D^2}{100\,l}\right)$, and $\Sigma\left(\frac{L\,D}{100\,l}\right)$, and substitute in (18)′, giving

$$x' = \frac{7.8 \times 0.25 - 5.5 \times 6.66}{6.66 \times 7.62 - (0.25)^2} = -0.68$$

$$y' = \frac{5.5 \times 0.25 - 7.8 \times 7.62}{6.66 \times 7.62 - (0.25)^2} = -1.15.$$

If the corrections to the latitudes and departures only are required, as in computing areas or co-ordinates, use (21)′ and (22)′. To do this, set x' on the slide rule and multiply by each number in the $\frac{L^2}{100\,l}$ column for the first term in each latitude correction, and by each $\frac{L\,D}{100\,l}$ for the first term in each departure correction. Then set to the value of y', and multiply by each $\frac{L\,D}{100\,l}$ for the second term in each latitude correction, and by each $\frac{D^2}{100\,l}$ for the second term in each departure correction, giving:

$v_{L_1} =$	$0.0 + 0.09 = +0.1$	$v_{D_1} =$	$+0.05 - 2.71 = -2.7$
$v_{L_2} =$	$-0.92 + 1.39 = +0.4$	$v_{D_2} =$	$+0.82 - 1.25 = -0.4$
$v_{L_3} =$	$-1.29 - 1.84 = -3.1$	$v_{D_3} =$	$-1.09 - 1.59 = -2.7$
$v_{L_4} =$	$0.0 + 0.07 = +0.1$	$v_{D_4} =$	$+0.04 - 2.10 = -2.0$
$v_{L_5} =$	$-2.97 + 0.0 = -3.0$	$v_{D_5} =$	$0.00 + 0.0 = 0.0$
Check, (23)′			
$\Sigma(v_L) = -q_1$	$= -5.5$	$\Sigma(v_D) = -q_2$	$= -7.8$

If the corrections to the distances only are required, as for a geometrically consistent description or record map, use (19)′; the x' setting of the slide rule giving all the first terms when multiplied by one-hundredth of the latitudes, and the y' setting the second terms when multiplied by one-hundredth of the departures. Thus:

$v_1 = +0.06$	-2.72	$= -2.66$
$v_2 = +1.23$	-1.88	$= -0.65$
$v_3 = +1.68$	$+2.40$	$= +4.08$
$v_4 = -0.04$	$+2.11$	$= +2.07$
$v_5 = -2.97$	$+0.0$	$= -2.97$
$\Sigma(v) = -0.68 \times 0.055 - 1.15 \times 0.078 = -0.13$ by (20)′		

If corrected co-ordinates and distances are both required, it will be less work to find the distance corrections first and the co-ordinate corrections from these by the first portions of (21)′ and (22)′.

TABLE No. 3.—PROBLEM FROM ACTUAL SURVEY.

Angles closed without adjustment.

Station.	Bearing.	Distance.	Latitude, L.		Departure, D.		L^2 / 100 l	D^2 / 100 l	$L\,D$ / 100 l
			+	−	+	−			
1.....	N. 44° 38.5′ E.	287.24	204.37	.....	201.83	.....	1.45	1.42	1.44
2.....	N. 52 42.5 E.	451.75	273.70	.....	359.40	.....	1.65	2.85	2.18
3.....	S. 2 00 W.	921.60	.....	321.04	.....	32.16	9.20	0.01	0.32
4.....	N. 88 34 W.	212.00	5.30	.....	.....	211.93	0.00	2.11	−0.05
5.....	S. 88 50.5 W.	317.30	.....	6.41	.....	317.24	0.01	3.17	0.06
6.....	North.....	443.60	443.60	.....	.....	.....	4.44	0.00	0.00
	Totals.....	.....	926.97	927.45	561.23	561.33	16.75	9.56	3.95
				926.97		561.23			
			q_1 =	− 0.48	q_2 =	− 0.10			

$$x' = \frac{-0.10 \times 3.95 + 0.48 \times 9.56}{160 - 15.55} = +0.029$$

$$y' = \frac{-0.48 \times 3.95 + 0.10 \times 16.75}{160 - 15.55} = -0.001$$

$v_1 = +0.05$; $v_{L_1} = +0.04$; $v_{D_1} = +0.04$

$v_2 = +0.08$; $v_{L_2} = +0.05$; $v_{D_2} = +0.06$

$v_3 = -0.26$; $v_{L_3} = +0.26$; $v_{D_3} = +0.01$

$v_4 = 0.00$; $v_{L_4} = 0.00$; $v_{D_4} = 0.00$

$v_5 = 0.00$; $v_{L_5} = 0.00$; $v_{D_5} = 0.00$

$v_6 = +0.13$; $v_{L_6} = +0.13$; $v_{D_6} = 0.00$

$\Sigma(v) = 0.00$; $\Sigma(v_L) = +0.48 = -q_1$; $\Sigma(v_D) = +0.11 = -q_2$

In some cases base or reference lines and stations are carefully located, and the other work adjusted to them; in others, a survey is extended after an adjustment has been made which it is desired not to disturb. These cases are treated by omitting the terms and corrections for the perfect lines, which forces all the corrections into the unadjusted parts.

Thus, in the foregoing problem, if 6–1 be regarded as perfect,

$$\Sigma\left(\frac{L^2}{100\,l}\right) = 12.31;\ \Sigma\left(\frac{D^2}{100\,l}\right) = 9.56;\ \Sigma\left(\frac{L\,D}{100\,l}\right) = 3.95.$$

$$x = \frac{-0.10 \times 3.95 + 0.48 \times 9.56}{9.56 \times 12.31 - (3.95)^2} = +0.041$$

$$y = \frac{-0.48 \times 3.95 + 0.10 \times 12.31}{9.56 \times 12.31 - (3.95)^2} = -0.007$$

$v_1 = +0.07;$	$v_{L_1} = +0.05;$	$v_{D_1} = +0.05$
$v_2 = +0.09;$	$v_{L_2} = +0.05;$	$v_{D_2} = +0.07$
$v_3 = -0.38;$	$v_{L_3} = +0.38;$	$v_{D_3} = +0.01$
$v_4 = +0.01;$	$v_{L_4} = 0.00;$	$v_{D_4} = -0.01$
$v_5 = +0.02;$	$v_{L_5} = 0.00;$	$v_{D_5} = -0.02$
$\Sigma(v) = -0.19;$	$\Sigma(v_L) = +0.48;$	$\Sigma(v_D) = +0.10$

By (20)′, $\Sigma(v) = -0.041 \times 4.44 + 0.007 \times 0.001 = -0.18$, for a check.

The method thus lends itself readily to the fitting of new to old work which has already been adjusted.

The application of the method of adjustment is given in considerable detail so that it can be used mechanically the same as the old compass survey method without looking up its theoretical basis. It cannot be stated so concisely as the old, and the numerical work involves more labor if corrected co-ordinates only are required. In case corrected distances are required, to give with the bearings geometrically consistent data for a record map or description, this requires less labor than the old method, for the corrected distances are used directly with the bearings, while in the old method neither corrected distances nor bearings can be obtained without a special computation after the co-ordinates have been adjusted.

The method has a good theoretical basis for the conditions assumed, viz.: small errors of closure for the angles and an inaccuracy of chaining increasing as the square root of the distance, while the old method has none for transit work.

DISCUSSION.

GEORGE W. TUTTLE, Esq. (by letter).—It is stated on page 454 that the most probable corrections will be those the sum of whose squares is a minimum after each correction has been divided by its probable error. It is the square of the probable error, however, that should be and has been used in the development of the formulas. Mr. Tuttle.

It would be inferred by a reader of the paper that Dr. Bowditch developed his method from least-square considerations, although such is not the case. In *The Analyst*, Vol. 1, No. 4, 1808, Dr. Bowditch presented his well-known method, in answer to a question proposed in a previous number of that journal, as to the best method of correcting a compass survey. Dr. Bowditch did not give the solution in mathematical form, but presented the now common graphical one, stating, in substance, that it was reasonable to assume that the corrections should be proportional to the length of the lines, and parallel to the closing side.

It remained for Dr. Adrian, the editor of *The Analyst*, in the same number, to demonstrate Dr. Bowditch's method by least squares, and this work is remarkable as the earliest application of the method of least squares to geodetic work, as well as for the general and instructive form in which it is given. This problem, and its solution by Dr. Adrian, forms an important and interesting part of the early history of mathematics in the United States. Dr. Adrian demonstrates the following formulas, which are put into the notation of the present paper:

$$\left.\begin{array}{l} x\,(\Sigma\, M \cos.^2 B + p^2\, \Sigma\, M \sin.^2 B) + y\,(1-p^2) \\ \Sigma\, M \sin.\, B \cos.\, B + q_1 = 0 \end{array}\right\} \ldots\ldots\ldots (1)$$

$$\left.\begin{array}{l} y\,(\Sigma\, M \sin.^2 B + p^2\, \Sigma\, M \cos.^2 B) + x\,(1-p^2) \\ \Sigma\, M \sin.\, B \cos.\, B + q_2 = 0 \end{array}\right\} \ldots\ldots\ldots (2)$$

$$\text{or, } x\left[\Sigma\left(\frac{L^2}{l}\right) + p^2\,\Sigma\left(\frac{D^2}{l}\right)\right] + \Sigma\left(\frac{L\,D}{l}\right)(1-p^2)\,y + q_1 = 0 \ldots (3)$$

$$x\,\Sigma\left(\frac{L\,D}{l}\right)(1-p^2) + \left[\Sigma\left(\frac{D^2}{l}\right) + p^2\,\Sigma\left(\frac{L^2}{l}\right)\right] y = 0 \ldots\ldots (4)$$

$$\text{where } p = \frac{\text{Probable error in bearing} \times \text{length of line.}}{\text{Probable error in length of line.}}$$

Equations (3) and (4) are more general forms of the author's Equations (17), which are obtained by making $p = 0$. Dr. Adrian notes, first, that when $p = 1$,

$x = \dfrac{-\,q_1}{l_1 + l_2 + l_3 + \ldots}$ and $y\,\dfrac{-\,q_2}{l_1 + l_2 + l_3 + \ldots}$, then $V_{L_1} = x\,l_1$, $V_{D_1} = y\,l_1$ and motion of angle points is in the direction of closing line. Second, when $p = 0$, motion of angle points is in the direction of l_1, l_2, l_3. . . . . .

Mr. Tuttle. Third, when $p = \alpha$, motion of angle points is in a direction perpendicular to the distances l_1, l_2, l_3, ...

It will be seen that the first case is the method of Dr. Bowditch; the second is that of the author, and supposes all the errors distributed among the sides only, the bearings remaining unchanged. The third solution takes up all the errors of the traverse in the bearings, and the measured distances remain unchanged.

Now, the foregoing general equations can be used with substantially the same amount of work as those proposed by the author, and with the use of his forms of computation. They have the advantage that the errors can be distributed among the sides and angles in any proportion, according to the relative accuracy of the measured sides and angles; these formulas have the defect however, that the correction to the bearing of any line is inversely proportional to the square root of the length of that line, while in a transit survey, where the lines are not excessively short, it should be independent of the distance.

The fundamental defect in the formula proposed by the author is, that it requires the errors to be distributed in the sides only, while the angular error, where many angles are involved, is often greater. When the errors have to be taken up, either in sides or angles alone, the corrections will, in general, be double what they would be if distributed through both.

That which follows refers largely to city work, with transits of good make, and reading to 20 seconds, and tapes having spring balance and temperature adjustment.

The probable error in chaining appears to increase less rapidly than the distance, and faster than the square root of the distance as assumed in the foregoing formulas. The law of the square root would probably be true if all errors to which distance measurements were subject were compensating, and made up of a large number of purely accidental errors, an equal number of positive and negative ones occurring. But as most of them are not compensating, and, as for any one line measured in a brief period, temperature and other errors proportional to distance may be considerable and cumulative, it would seem to be quite as accurate to take the error of chaining as being proportional to the distance, in which form the error is usually stated.

It would seem that for an accurate and economical survey, some relation should exist between the accuracy demanded of the chain, and that of the transit. No adjustment that can be devised is so perfect or satisfactory that it can take the place of accurate work. It is quite important, therefore, that the proper relation between the precision of the line and angle measurements should be determined.

Were the angle and distance measurements equally difficult, for the sake of economy, the allowable error should be divided equally between them, as Dr. Bowditch's method assumes. The angular measurements

being usually less difficult, some greater precision may be observed in their measurement, but the respective errors, usually, should not depart widely from equality, and in this particular it appears desirable that the assumption implied in the usual rule, *i. e.*, that errors due to chaining and angle measurements are equally probable, should obtain in practice. Mr. Tuttle.

Assume a closed traverse of n sides and angles, with an angular error of closure of $a\sqrt{n}$, where a = allowable error in measurement of one angle; then, to make the figure geometrically consistent, as far as the angles are concerned, a correction of $\frac{a\sqrt{n}}{n}$ should be applied to each angle. This is the same, in effect, as assuming the bearing of one line in the traverse, from which the bearings of the other sides are derived, and weighting the two determinations of the bearing of any given side found in passing from the assumed line, round to the right and left, respectively, inversely as the number of angles passed through.

Let m be the value of any bearing determined by passing through z angles to the right, and m_1 by passing through $(n - z)$ angles to the left, and m_m and r_m the value and probable error of bearing, after the error of closure is applied.

Then,
$$m_m = \frac{\frac{m}{r^2} + \frac{m_1}{r_1^2}}{\frac{1}{r^2} + \frac{1}{r_1^2}}$$

where r and r_1 represent the probable error of determination in passing round the closed figure to the right and left, respectively.

It may be observed that the probable error, as here used, is that obtained from the allowable difference of angular closure, *i. e.*, $a\sqrt{n}$, and is consequently the probable error corresponding to this difference, and the maximum value which the probable error should have.

Now,
$$\frac{1}{r^2} = \frac{1}{(0.674a)^2 z}, \text{ and } \frac{1}{r_1^2} = \frac{1}{(0.674a)^2 (n - z)},$$

so that
$$m_m = \frac{\frac{m}{z} + \frac{m_1}{n - z}}{\frac{1}{z} + \frac{1}{n - z}} = \frac{m(n - z) + m_1 z}{n} \quad \ldots\ldots\ldots\ldots (5)$$

and
$$r_m = \frac{1}{\sqrt{\frac{1}{r^2} + \frac{1}{r_1^2}}} = \frac{0.674a}{\sqrt{\frac{1}{z} + \frac{1}{n - z}}} = 0.674a\sqrt{\frac{n z - z^2}{n}} \quad \ldots\ldots (6)$$

From the foregoing, it is seen that the probable error of the bearing distant $z = \frac{n}{2}$ from the assumed side is $r_{m\left(z = \frac{n}{2}\right)} = 0.34a\sqrt{n}$,

Mr. Tuttle. while the probable error of a single determination is $r_{\left(z=\frac{n}{2}\right)} = 0.47a\sqrt{n}$. The allowable error, $a\sqrt{n}$, is, therefore, 2.1 times the probable error of a single determination as here defined and used.

It is easily seen from Equation (6) that the probable errors of the corrected bearings vary from zero at the base to $0.34\ a\sqrt{n}$ at the bearing $\frac{n}{2}$ angles distant from the base, in proportion to the ordinate of a circle of diameter n, divided into segments z and $n - z$.

As it is impracticable to assign different probable errors to bearings in the determination of corrections, or to measure the angles so as to secure bearings having a uniform probable error, the square root of the mean value of the squares of the probable errors should be taken as the constant probable error of an angle; this is equivalent to making p^2, in Equations (3) and (4) constant, and equal to its mean value. We will define the square root of the mean of the squares of the probable errors as the mean probable error. The mean probable error of bearing, in a traverse of n angles, will therefore be, approximately,

$$0.674a\sqrt{\frac{1}{n}\int_0^{\frac{n}{2}}\left(\frac{n\,z - z^2}{n}\right)dz} = 0.275\ a\sqrt{n},$$

and the mean value of the allowable error, being 2.1 times the latter, will equal $0.58\ a\sqrt{n}$. While this is the mean value of the allowable error in traverse work connected with fixed bearings, it is the actual allowable error in a closed circuit where no bearings are fixed, since each bearing is similarly circumstanced relatively to all the others.

From the record of the angles measured with several good 20-second transits, under service conditions, and by different observers, the probable error of a single angle measurement was found to be less than 12 seconds. This quantity, therefore, is taken as the maximum probable error of a single measurement, and consequently, it is found that 12 seconds $\times$ 2.1 = 25.0 seconds is the allowable error or difference between measurements.

The allowable error of chaining with the tape before described has been found to be about 1 in 10 000, and an angular error of 20.6 seconds will cause an error of an equal amount in the location of the end of any given line. Therefore, when the mean allowable error of a traverse equals 20.6 seconds, the error due to angular measurement will be approximately equal to the error due to chaining, when measured with the accuracy of 1 in 10 000. We have already found the mean allowable error in a traverse of n sides to be $0.58\ a\sqrt{n}$; therefore, to find the value of a for equal precision in angles and chaining, we have the equation $0.58\ a\sqrt{n} = 20.6$ seconds.

Table No. 4 has been prepared from this equation. The first column gives values of n, the number of angles in the traverse. The second, the values of a, the allowable error in angular measurement, *i. e.*, the difference between two measurements should not exceed this amount for equal precision in sides and angles. The third column gives the approximate number of repetitions of angle necessary for equal precision in sides and angles when $p = 10\,000 \times 0.58\, a \sqrt{n}$ arc 1 second $= 1$. Mr. Tuttle.

TABLE No. 4.

n.	a.	Repetitions necessary.
4	18	2
9	12	3
16	9	4
25	7	6
36	6	8

From a considerable experience in adjusting careful traverse work to triangulation, it has been found that where many angles are involved, the divergence due to angular errors may be considerably greater than errors due to chaining, and in this way the foregoing deductions as to the necessary care in measuring traverses of many angles has been verified.

Equations (1), (2), (3) and (4) may be easily modified to make the probable error in distance measurement proportional to the length of line, and the probable error in bearings independent of the same. The equations then take the form:

$$\left.\begin{array}{l} x\,[\Sigma\ M^2 \cos.^2 B + p^2\ \Sigma\ M^2 \sin.^2 B] + y\,(1-p^2) \\ M^2 \sin.\ B \cos.\ B + q_1 = 0 \end{array}\right\} \quad \ldots\ldots(7)$$

$$\left.\begin{array}{l} y\,[\Sigma\ M^2 \sin.\ B + p^2\ \Sigma\ M^2 \cos.\ B] + x\,(1-p^2) \\ M^2 \sin.\ B\ \cos.\ B + q_2 = 0 \end{array}\right\} \quad \ldots\ldots. (8)$$

or,

$$x\,[\Sigma\ L^2 + p^2\ \Sigma\ D^2] + \Sigma\ L\ D\,(1-p^2)\,y + q_1 = 0 \ldots\ldots(9)$$

$$x\,(1-p^2)\ \Sigma\ L\ D + y\,[\Sigma\ D^2 + p^2\ \Sigma\ L^2] + q_2 = 0 \ldots\ldots(10)$$

Now, when the angles and sides are equally well measured, p will equal unity, and we have

$$x = \frac{-q_1}{l_1^2 + l_2^2 + l_3^2 + \ldots.} \qquad V_{L_1} = l_1^2\, x \ldots\ldots\ldots\ldots(11)$$

$$y = \frac{-q_2}{l_1^2 + l_2^4 + l_3^2 + \ldots.} \qquad V_{D_1} = l_1^2\, y \ldots\ldots\ldots\ldots(12)$$

It will be seen that in these equations the squares of the sides are used in precisely the same way that the sides are used in the common formulas. While the latter make the corrections to be applied to long lines comparatively slight and the corrections to short ones large;

Mr. Tuttle. Formulas (11) and (12) make the corrections to the location of any given point less when passing along a short side, and greater in the case of a long side, by assuming the probable error in the location of a given point as proportional to the distance to that point, instead of to the square root of the distance.

These equations are so simple, and so readily deduced, that the writer has some misgivings in presenting them as new, although he has no knowledge that they have ever been proposed. They appear to the writer to be superior to the common formulas for the adjustment of transit work, and also have the merit of simplicity.

Mr. Fuertes. JAMES H. FUERTES, M. Am. Soc. C. E.—During the past two years, in the investigations connected with the Philadelphia Water Supply Improvement, for the Mayor's expert Water Commission, and the New York City Water Supply investigations, for the New York Merchants' Association, the speaker has conducted rapid surveys over a large area of mountainous country, for the purpose of securing data relative to stream flow and storage facilities. A short account of the methods used and the results obtained may be of interest.

For the New York investigations the work consisted of reconnaissances for aqueduct lines, and surveys of reservoir sites and water-shed limits, in the Adirondack, Catskill, Kittatinny and Shawangunk regions, for which no topographical maps were available. As the time allowed was very short, quick approximate methods were necessarily used.

In the Catskills the information desired was the location of suitable dam sites, the capacities of the reservoirs formed by these dams, and the area tributary to each, in order to arrive at conclusions as to the available daily yield of this water-shed. It was, at first, a rather disheartening task; favorable locations for masonry dams appearing only at the lowest sites on each of the three main streams, the Esopus, Catskill and Schoharie Creeks, and the country being so rough and broken that progress was necessarily slow.

As a foundation for the work, the best available maps were secured. These were on so small a scale as to be of little use except for the purpose of estimating water-shed areas. The equipment of a party consisted of a pocket aneroid barometer, a pocket compass, a hand-level, and a horse and buggy. Each party consisted of three persons, two for the field and one to remain at headquarters and record at short intervals the readings of the stationary barometer, noting the exact time of each reading. Readings were sometimes taken at about 3-minute intervals, when the fluctuations in pressure were rapid. The time was noted at each reading, both in the field and at headquarters. At night the readings of the stationary barometer were plotted and the proper corrections of the field readings were made. Sometimes it was necessary to compensate for lapse of time, when an area of low pressure was moving toward or away from the

observers; this was done by shifting the time scale. At times the changes in the barometer would be as much as 300 to 400 ft. in a few hours, but by taking several readings at consecutive stations the effects of such variations could be eliminated. Generally, the barometric readings, after correction, could be depended upon within about 10 ft. Mr. Fuertes.

The field party first meandered all the roads in, and, crossing, the creek valleys, taking bearings at every change of direction, and elevations at every angle point, and at points where the slopes changed. The distances from point to point were measured by counting the revolutions of the buggy wheel. The corrections for the slip of the wheel, according as the roads were smooth or rough, frozen, icy, muddy, stiff, hilly or flat, were determined each day by plotting the day's work and checking the distances by counting the rails of the railroad track between road crossings. Where possible, surveys were made in complete circuits, so that distances and directions could be checked from two independent lines. After the main lines through the valleys were completed, cross-sections were taken at various points where the topography indicated that such sections were desirable.

Elevations were checked by reading the barometer at the different stations every time points were passed where elevations had been taken on previous days. The elevations of the railroad stations were assumed as starting points for the different lines. These were taken from the time table of the Ulster & Delaware Railroad, or from the "Dictionary of Altitudes" published by the U. S. Geological Survey. At Cold Brook Station, on the Ulster & Delaware Railroad, the barometer indicated a discrepancy of about 40 ft. in the elevations, as compared with the time table elevation. Every time this station was passed the same difference of elevation was found; it was therefore checked, first with a hand-level, and afterward with a wye-level, and the barometric elevation was found to be substantially correct.

Each day's work was plotted at night, the contours at 20-ft. intervals, roads, buildings, railroads, bridges, etc., being put on the sheets, which were then sent to the New York office to be worked into record shape. The sheets for the general map were plotted to a scale of about 1 mile to an inch, and the detailed maps of the creek valleys, on a scale of 500 ft. to an inch. The plotting generally checked out quite closely; closely enough to enable the probable storage capacity of the reservoirs to be estimated with a sufficient degree of accuracy. It was found that the error was generally on the side of safety; that is, the tendency was to underestimate distances rather than to overestimate them.

The total cost of the field work for the Esopus Valley, including all salaries, team hire, hotel bills and traveling expenses, was $457; the cost of making the record maps was $136; or, a total of $593 for the

Mr. Fuertes. survey and maps complete. The territory embraced in this survey, over the whole of which contours were taken at 20-ft. intervals, covered 9 500 acres; thus, the cost of the field work was at the rate of 4.8 cents per acre, and the cost of the record maps, on a scale of 500 ft. to an inch, about 1.5 cents per acre, making the total cost 6.3 cents per acre.

The reliability of the barometers was tested most severely in making the survey of the "Drowned Lands" in the Wallkill Valley. Just south of the crossing of the Erie Railroad over the Wallkill River, the valley widens out and forms a broad plain from one to five miles in width. This plain is nearly flat, both lengthwise and transversely, and extends well down into New Jersey. It was at one time, undoubtedly, the bottom of a lake. During the winter and spring the whole valley frequently lies several feet under water. In getting elevations in this valley care was exercised to take as many barometric readings as possible on points where elevations had been taken previously. Elevations were carried down from Goshen and Middletown. After the completion of the work, comparisons were made with the Geological Survey sheets of New Jersey, and it was found that the contours carried down from the Erie Railroad crossing fitted those brought up from tide-water through New Jersey.

In the case of the erroneous elevation found at Cold Brook Station, on the Ulster & Delaware Railroad, the attention of the Railroad Company was not called to the discrepancy, because the elevation was taken from the time-table, and was not official. The valley through which the railroad runs is quite beautiful, and is much frequented by tourists and summer visitors, and one of the features of the time-table was the publication of the elevations of the railroad stations. The profile of the railroad was not secured. At the time, it was thought that the error might have been simply an oversight in proof-reading. The elevation is given correctly, on the authority of the railroad in the "Dictionary of Altitudes."

Mr. Luster. W. H. LUSTER, Jr., Assoc. M. Am. Soc. C. E.—The speaker has done quite an amount of topographic work, with both aneroid and wye-level, and probably did some of the work near the New Jersey State line to which Mr. Fuertes refers.

Part of this topographical work was in a rough section of the Adirondacks, where the general method followed was to run lines of levels along the railroads and country highways. These levels were used as the base for all stadia and aneroid work, vertical angles being used on all stadia lines.

In taking the topography of a township the lines were first run around the tract, using transit and steel tape, the horizontal and vertical angles were read and stadia distances recorded; the latter was reduced to the horizontal and used to check the tape distance.

A township contains 36 sq. miles and measures 24 miles around the border. The closure of the horizontal work was 30 ft. and of the vertical about 6 ft. Mr. Luster.

This work was all cross-checked by running lines on the subdivisions of the township, sometimes by transit and tape and at other times with compass and stadia, according to the importance of the line. These cross-lines always crossed a line of levels, so that perfect vertical control was secured at all times. The azimuth was checked by sun observations and triangulation.

All horizontal measurements were taken between vertical rods which were driven solidly in the ground and made vertical by the use of two attached levels. Down the center of these rods there was a slot which just allowed the entrance of a projection on a plate attached to the zero end of the tape. The tape was made horizontal by the use of a hand-level, and the distance between the centers of the slots was then read and recorded.

Some of the distances taped over had slopes of 45 degrees. Taking that fact into consideration with the distance measured, 24 miles, the closure of 30 ft. was remarkably good.

It is a peculiar fact that in all the township line work which the speaker has done in the Adirondacks, he has found that with few exceptions these lines cross the summits of all the high peaks or very close to them, and in almost every instance a triangulation station can be located on the line which will give perfect view to other stations.

At these points a small signal was erected and a round of angles measured, and at a later date it was observed on from the other stations. These triangulations were made with a 20-second Buff and Berger transit. The results of these triangulations were plotted on a projection and the lines then adjusted to these points graphically. The scale of this map was 1 in 10 000, which made the perceptible error on the map very slight.

The topography was filled in by aneroid, which, for ground of this nature, was a rapid and fairly accurate way of working. Each township was divided into lots, the lines of which were run by compass and stadia, the vertical angles being read for the vertical control. From these lines a pacing traverse was run to the interior of each lot, and aneroid readings taken at all points of topographic interest. The remaining contours were sketched in by eye. The aneroids were read at the start and finish of each traverse, and the points taken in the interior were adjusted in proportion to the elapsed time. Readings for comparison were taken at the camp during the day by the man in charge, if his other duties did not prevent; as this often happened, these readings were little used and dependence was placed on average readings taken on the same point. These aneroid

Mr. Luster. heights can be depended on within 10 ft., five or six check readings were taken on a point of known elevation, which led the speaker to take the error mentioned as a maximum.

The speed of such a survey, when all hands, fifteen men, were working on the line, was about 2 miles per day.

The foregoing method differs from that carried on in New Jersey. In that State the ground is not so rough and is not heavily timbered, so that the various features of the ground can be readily seen and no useless lines need be run. For country of this nature the wye-level was used for all topography. In the few cases where the country was hilly and timbered aneroids were depended on, but were used on only very short lines.

Formulas were never used to distribute the error, as the number of stations involved would have made the task so great that the results of the work would have been greatly delayed before publication, and, in the speaker's opinion, the scale of a map, where so small as that mentioned, does not warrant such refinement.

The paper has been read carefully, and, while admitting that a method of adjustment is necessary, the one given seems to the speaker to be too cumbersome for practical use. In making an adjustment of a survey the speaker uses the following method: The lines which can be measured without any difficulty are taken as correct and the difference then remaining is proportioned to the other lines in proportion to their lengths. If the ground is open and the located corners can be seen from two or more points in the field, a base line is carefully measured, from the ends of which these corners can be seen; the angles are measured and then the sides are computed to check the measured distances. It has been the speaker's experience that by this method very little error will be found in lines easily measured, and no refinements need be used to adjust this small difference.

Mr. Lee. W. B. LEE, M. Am. Soc. C. E.—The speaker has often used Dr. Bowditch's method of adjusting a survey, probably for the lack of something better, and has fallen into the error, common to those who have applied it to a transit survey, namely, the giving of insufficient credit to the accuracy of the angle measurements.

No survey should be accepted, the original notes of which are not sufficiently accurate within their limits, which may be anywhere between those of mountain land and city lots. This condition being satisfied, no surveyor can afford to make, nor will any owner consent to pay for, an elaborate correction. Surveys are made for the purpose of locating fixed points or corners. If the bearings and distances when re-run will not locate these points accurately, it matters not whether the error be one of bearing or distance.

Since the present practice tends to an accuracy of instrument work greater than that of chaining, the speaker would recommend that

these two be adjusted to a common basis before a survey is made. Mr. Lee.
Then Dr. Bowditch's simple and rapid method of correction may be applied without detriment to either.

Much depends upon the circumstances attending the making of a survey; that is, the conditions under which it is made, and the nature of the territory. Here arise abundant opportunities for the exercise of judgment.

The speaker recently made a survey of an industrial plant which was scattered along a railway for about half a mile, making a circuit then, of about a mile. The line along the railway was assumed as a meridian and its length as correct; then the meandering line behind the plant was adjusted to it. Numerous shots and measurements to corners of buildings and other fixed points proved the accuracy of the work and the rationality of the adjustment.